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Canonical formulation of $N = 2$ supergravity in terms of the Ashtekar variable

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Abstract

We reconstruct the Ashtekar's canonical formulation of $N = 2$ supergravity (SUGRA) starting from the $N = 2$ chiral Lagrangian derived by closely following the method employed in the usual SUGRA. In order to get the full graded algebra of the Gauss, $U(1)$ gauge and right-handed supersymmetry (SUSY) constraints, we extend the internal, global $O(2)$ invariance to local one by introducing a cosmological constant to the chiral Lagrangian. The resultant Lagrangian does not contain any auxiliary fields in contrast with the 2-form SUGRA and the SUSY transformation parameters are not constrained at all. We derive the canonical formulation of the $N = 2$ theory in such a manner as the relation with the usual SUGRA be explicit at least in classical level, and show that the algebra of the Gauss, $U(1)$ gauge and right-handed SUSY constraints form the graded algebra, $G^2SU(2)$. Furthermore, we introduce the graded variables associated with the $G^2SU(2)$ algebra and we rewrite the canonical constraints in a simple form in terms of these variables. We quantize the theory in the graded-connection representation and discuss the solutions of quantum constraints.

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1. Introduction

The nonperturbative canonical treatment of supergravity (SUGRA) in terms of the Ashtekar variable [1] was firstly discussed about the simplest $N = 1$ theory in [2]. In this theory the chiral Lagrangian was constructed by using the self-dual connection which couples to only a right-handed spin-3/2 field, and this Lagrangian has two kinds of right- and left-handed supersymmetry (SUSY) invariances in the first-order formalism. Therefore two types of the SUSY constraints, which generate those SUSY transformations, appear in the canonical formulation. Fülöp [3] and Armand-Ugon *et al.* [4] showed that in $N = 1$ chiral SUGRA the $SU(2)$ algebra generated by the Gauss-law constraint is graded by means of the right-handed SUSY constraint. All the constraints were also rewritten in a simple form in [4] towards a loop representation of quantum canonical SUGRA, by using graded connection and momentum variables associated with the graded algebra which is called the $GSU(2)$ algebra [5].¹ Furthermore the spin network states [6] of SUGRA was recently constructed in [7] based on the representation of this graded algebra.

The extension of the Ashtekar's canonical formulation to $N = 2$ extended SUGRA was mainly developed in the context of the 2-form gravity [8, 9, 10]. The chiral Lagrangian of $N = 2$ SUGRA was constructed in [8, 9] based on the $N = 1$ 2-form SUGRA [11] with auxiliary fields which are needed to write the chiral (2-form) Lagrangian: It was proved that the SUSY algebra is not closed at the level of transformation algebra on auxiliary fields, but actually closes at the level of the canonical formulation. On the other hand, in [10] the canonical formulation of the BF theory as a topological field theory [12] was derived for an appropriate graded

¹In Ref. [7] it is pointed out that the algebra of $GSU(2)$ corresponds to the super Lie algebra, $Osp(1/2)$.

algebra of $SU(2)$ (which henceforth will be referred to as $G^2SU(2)$ ² following [10]), and it was shown that the $G^2SU(2)$ BF theory subject to some algebraic constraints can be cast into the $N = 2$ 2-form SUGRA.

In this paper we reconstruct the Ashtekar's canonical formulation of $N = 2$ SUGRA starting from the $N = 2$ chiral Lagrangian derived by closely following the method employed in the usual SUGRA. In Sec. 2 we present the globally $O(2)$ invariant Lagrangian of $N = 2$ chiral SUGRA slightly modified from the Lagrangian in [14]. In order to get the full graded algebra of the Gauss, $U(1)$ gauge and right-handed SUSY constraints, we extend in Sec. 3 the internal, global $O(2)$ invariance to local one by introducing a cosmological constant to the chiral Lagrangian. The resultant Lagrangian does not contain any auxiliary fields in contrast with the 2-form SUGRA and the SUSY transformation parameters are not constrained at all. In Sec. 4 we derive the canonical formulation of the $N = 2$ theory in such a manner as the relation with the usual SUGRA be explicit at least in classical level, and we show that the algebra of the Gauss, $U(1)$ gauge and right-handed SUSY constraints form the graded algebra, $G^2SU(2)$. In Sec. 5 we introduce the graded variables associated with the $G^2SU(2)$ algebra and rewrite the canonical constraints in a simple form in terms of these variables. We quantize the theory in the graded-connection representation and discuss the solutions of quantum constraints in Sec. 6. Our conclusions are included in Sec. 7.

2. The globally $O(2)$ invariant Lagrangian

Firstly we present the chiral Lagrangian of $N = 2$ SUGRA constructed in [14]. The independent variables are a tetrad e^i_μ , two (Majorana) Rarita-Schwinger fields

² The algebra of $G^2SU(2)$ corresponds to the super Lie algebra, $Osp(2/2)$ [13].

$\psi_\mu^{(I)}$, a Maxwell field A_μ and a (complex) self-dual connection $A_{ij\mu}^{(+)}$ which satisfies $(1/2)\epsilon_{ij}^{kl}A_{kl\mu}^{(+)} = iA_{ij\mu}^{(+)}$.³ The $N = 2$ chiral Lagrangian density in terms of these variables is written in first-order form as

$$\begin{aligned}\mathcal{L}_{N=2}^{(+)} = & -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}e_\mu^ie_\nu^jR_{ij\rho\sigma}^{(+)} - \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{R\mu}^{(I)}\gamma_\rho D_\sigma^{(+)}\psi_{R\nu}^{(I)} - \frac{e}{2}(F_{\mu\nu}^{(-)})^2 \\ & + \frac{1}{4\sqrt{2}}\bar{\psi}_\mu^{(I)}\{e(F^{\mu\nu} + \hat{F}^{\mu\nu}) + i\gamma_5(\tilde{F}^{\mu\nu} + \tilde{\hat{F}}^{\mu\nu})\}\psi_\nu^{(J)}\epsilon^{(I)(J)} \\ & + \frac{i}{8}\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}_{L\mu}^{(I)}\psi_{R\nu}^{(J)})\bar{\psi}_{R\rho}^{(K)}\psi_{L\sigma}^{(L)}\epsilon^{(I)(J)}\epsilon^{(K)(L)},\end{aligned}\quad (2.1)$$

which is globally $O(2)$ invariant. Here e denotes $\det(e_\mu^i)$, $\epsilon^{(I)(J)} = -\epsilon^{(J)(I)}$ and $F^{(-)\mu\nu} := (1/2)(F^{\mu\nu} + ie^{-1}\tilde{F}^{\mu\nu})$ with $\tilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. The covariant derivative $D_\mu^{(+)}$ and the curvature $R^{(+)\,ij}_{\mu\nu}$ are

$$\begin{aligned}D_\mu^{(+)} &:= \partial_\mu + \frac{i}{2}A_{ij\mu}^{(+)}S^{ij}, \\ R^{(+)\,ij}_{\mu\nu} &:= 2(\partial_{[\mu}A^{(+)\,ij}_{\nu]} + A^{(+)\,i}_{k[\mu}A^{(+)\,kj}_{\nu]}),\end{aligned}\quad (2.2)$$

while $\hat{F}_{\mu\nu}$ in the second line of (2.1) is defined as

$$\hat{F}_{\mu\nu} := F_{\mu\nu} - \frac{1}{\sqrt{2}}\bar{\psi}_\mu^{(I)}\psi_\nu^{(J)}\epsilon^{(I)(J)}.\quad (2.3)$$

Note that we have used $(F_{\mu\nu}^{(-)})^2$ as the Maxwell kinetic term in Eq. (2.1), which allows us to rewrite the canonical constraints in terms of the graded variables associated with the graded algebra, $G^2SU(2)$, as will be explained later. In this respect the chiral Lagrangian of Eq. (2.1) differs from that constructed in [14].

The last four-fermion contact term in Eq. (2.1) is pure imaginary but this term is necessary to reproduce the Lagrangian of the usual $N = 2$ SUGRA in the second-order formalism. Indeed, if we solve the equation $\delta\mathcal{L}_{N=2}^{(+)}/\delta A^{(+)} = 0$ with respect to

³ Greek letters μ, ν, \dots , are spacetime indices, Lattin letters i, j, \dots , are local Lorentz indices and $(I), (J), \dots (= (1), (2))$, denote $O(2)$ internal indices. We take the Minkowski metric $\eta_{ij} = \text{diag}(-1, +1, +1, +1)$ and the totally antisymmetric tensor ϵ_{ijkl} is normalized as $\epsilon_{0123} = +1$. We define $\epsilon_{\mu\nu\rho\sigma}$ and $\epsilon^{\mu\nu\rho\sigma}$ as tensor densities which take values of $+1$ or -1 .

$A_{ij\mu}^{(+)}$ and use the obtained solution in the first two terms in Eq. (2.1), then those terms give rise to a number of four-fermion contact terms, which are complex with the imaginary term being written as

$$\frac{i}{8}\epsilon^{\mu\nu\rho\sigma}T_{\lambda\mu\nu}T^\lambda{}_{\rho\sigma} = -\frac{i}{16}\epsilon^{\mu\nu\rho\sigma}(\overline{\psi}_{R\mu}^{(I)}\gamma_\lambda\psi_{R\nu}^{(K)})\overline{\psi}_{R\rho}^{(J)}\gamma^\lambda\psi_{R\sigma}^{(L)}\epsilon^{(I)(J)}\epsilon^{(K)(L)}, \quad (2.4)$$

where the torsion tensor is defined by $T^i{}_{\mu\nu} = -2D_{[\mu}e_{\nu]}^i$ with $D_\mu e_\nu^i = \partial_\mu e_\nu^i + A^i{}_{j\mu}e_\nu^j$.

The last term in Eq. (2.1), on the other hand, can be rewritten as

$$\begin{aligned} & \frac{i}{8}\epsilon^{\mu\nu\rho\sigma}(\overline{\psi}_{L\mu}^{(I)}\psi_{R\nu}^{(J)})\overline{\psi}_{R\rho}^{(K)}\psi_{L\sigma}^{(L)}\epsilon^{(I)(J)}\epsilon^{(K)(L)} \\ &= \frac{i}{16}\epsilon^{\mu\nu\rho\sigma}(\overline{\psi}_{R\mu}^{(I)}\gamma_\lambda\psi_{R\nu}^{(K)})\overline{\psi}_{R\rho}^{(J)}\gamma^\lambda\psi_{R\sigma}^{(L)}\epsilon^{(I)(J)}\epsilon^{(K)(L)} \end{aligned} \quad (2.5)$$

by using a Fierz transformation, and exactly cancels with the pure imaginary term of Eq. (2.4). Therefore the $\mathcal{L}_{N=2}^{(+)}$ [second order] of $N = 2$ chiral SUGRA is reduced to that of the usual one up to imaginary boundary terms; namely, we have

$$\begin{aligned} \mathcal{L}_{N=2}^{(+)}[\text{second order}] &= \mathcal{L}_{N=2 \text{ usual SUGRA}}[\text{second order}] \\ &\quad -\frac{1}{4}\partial_\mu\{\epsilon^{\mu\nu\rho\sigma}(\overline{\psi}_\nu^{(I)}\gamma_\rho\psi_\sigma^{(I)} + 2iA_\nu\partial_\rho A_\sigma)\}. \end{aligned} \quad (2.6)$$

Note that a boundary term quadratic in the Maxwell field A_μ appears in (2.6) since we choose $(F_{\mu\nu}^{(-)})^2$ as the kinetic term in Eq. (2.1).

3. Gauging the $O(2)$ invariance

The global $O(2)$ invariance of Eq. (2.1) can be gauged by introducing a minimal coupling for $\psi_\mu^{(I)}$ and A_μ , which automatically requires a spin-3/2 mass-like term and a cosmological term in the Lagrangian [15]. These three terms are written as

$$\begin{aligned} \mathcal{L}_{\text{cosm}} &= \frac{\lambda}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu^{(I)}\gamma_\rho\psi_\nu^{(J)}A_\sigma\epsilon^{(I)(J)} \\ &\quad -\sqrt{2}i\lambda e\overline{\psi}_\mu^{(I)}S^{\mu\nu}\psi_\nu^{(I)} + 6\lambda^2 e \end{aligned} \quad (3.1)$$

with the gauge coupling constant λ . Here the cosmological constant Λ is related to λ as $\Lambda = -6\lambda^2$. Note that the first term of Eq. (3.1) is comparable with the kinetic term of $\psi_{R\mu}^{(I)}$ in Eq. (2.1), since this term can be rewritten as

$$\frac{\lambda}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu^{(I)}\gamma_\rho\psi_\nu^{(J)}A_\sigma\epsilon^{(I)(J)} = \lambda\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_{R\mu}^{(I)}\gamma_\rho\psi_{R\nu}^{(J)}A_\sigma\epsilon^{(I)(J)}. \quad (3.2)$$

We denote the chiral Lagrangian as the sum of Eqs. (2.1) and (3.1); namely,

$$\mathcal{L}^{(+)} := \mathcal{L}_{N=2}^{(+)} + \mathcal{L}_{\text{cosm}}. \quad (3.3)$$

Because of Eq. (2.6), the $\mathcal{L}^{(+)}$ of Eq.(3.3) in the second-order formalism is invariant under the SUSY transformation of the usual *gauged* $N = 2$ SUGRA [15] given by

$$\begin{aligned} \delta e_\mu^i &= i \overline{\alpha}^{(I)} \gamma^i \psi_\mu^{(I)}, \\ \delta A_\mu &= \sqrt{2} \epsilon^{(I)(J)} \overline{\alpha}^{(I)} \psi_\mu^{(J)}, \\ \delta \psi_\mu^{(I)} &= 2\{D_\mu[A(e, \psi^{(I)})]\alpha^{(I)} - \lambda\epsilon^{(I)(J)}A_\mu\alpha^{(J)}\} \\ &\quad + \frac{i}{\sqrt{2}} \epsilon^{(I)(J)} \left(\hat{F}_{\mu\nu} \gamma^\nu + \frac{i}{2} e \epsilon_{\mu\nu\rho\sigma} \hat{F}^{\rho\sigma} \gamma^\nu \gamma_5 \right) \alpha^{(J)} \\ &\quad - \sqrt{2}i \lambda \gamma_\mu \alpha^{(I)} \end{aligned} \quad (3.4)$$

with $A_{ij\mu}(e, \psi^{(I)})$ in $\delta\psi_\mu^{(I)}$ being defined as the sum of the Ricci rotation coefficients $A_{ij\mu}(e)$ and $K_{ij\mu}$ which is expressed as

$$K_{ij\mu} = \frac{i}{4}(\overline{\psi}_{[i}^{(I)} \gamma_{|\mu|} \psi_{j]}^{(I)} + \overline{\psi}_{[i}^{(I)} \gamma_{|j|} \psi_{\mu]}^{(I)} - \overline{\psi}_{[j}^{(I)} \gamma_{|i|} \psi_{\mu]}^{(I)}). \quad (3.5)$$

On the other hand, the first-order (i.e., “off-shell”) SUSY invariance of $\mathcal{L}^{(+)}$ may be realized by introducing the right- and left-handed SUSY transformations as in the case of $N = 1$ chiral SUGRA [2, 16].

4. The canonical formulation of $N = 2$ chiral SUGRA

Starting with the chiral Lagrangian $\mathcal{L}^{(+)}$ of Eq. (3.3), let us derive the canonical formulation of $N = 2$ chiral SUGRA by means of the (3+1) decomposition of space-time. For this purpose we assume that the topology of spacetime M is $\Sigma \times R$ for some three-manifold Σ so that a time coordinate function t is defined on M . Then the time component of the tetrad can be defined as ⁴

$$e_t^i = Nn^i + N^a e_a^i. \quad (4.1)$$

Here n^i is the timelike unit vector orthogonal to e_{ia} , i.e., $n^i e_{ia} = 0$ and $n^i n_i = -1$, while N and N^a denote the lapse function and the shift vector, respectively. Furthermore, we give a restriction on the tetrad with the choice $n_i = (-1, 0, 0, 0)$ in order to simplify the Legendre transform of Eq. (3.3). Once this choice is made, e_{Ia} becomes tangent to the constant t surfaces Σ and $e_{0a} = 0$. Therefore we change the notation e_{Ia} to E_{Ia} below. We also take the spatial restriction of the totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ as $\epsilon^{abc} := \epsilon_t^{abc}$, while $\epsilon^{IJK} := \epsilon_0^{IJK}$.

Under the above gauge condition of the tetrad, the (3+1) decomposition of Eq. (3.3) yields the kinetic terms, ⁵

$$\mathcal{L}_{\text{kin}}^{(+)} = \tilde{E}_I^a \dot{\mathcal{A}}_a^I - \tilde{\pi}^{(I)}{}_A{}^a \dot{\psi}^{(I)A}{}_a + {}^+ \tilde{\pi}^a \dot{A}_a, \quad (4.2)$$

⁴Latin letters a, b, \dots are the spatial part of the spacetime indices μ, ν, \dots , and capital letters I, J, \dots denote the spatial part of the local Lorentz indices i, j, \dots .

⁵ For convenience of calculation we use the two-component spinor notation in the canonical formulation. Two-component spinor indices A, B, \dots , and A', B', \dots , are raised and lowered with antisymmetric spinors $\epsilon^{AB}, \epsilon_{AB}$, and their conjugates $\epsilon^{A'B'}, \epsilon_{A'B'}$, such that $\psi^A = \epsilon^{AB} \psi_B$ and $\varphi_B = \varphi^A \epsilon_{AB}$. The Infeld-van der Waerden symbol $\sigma_{iAA'}$ are taken in this paper to be $(\sigma_0, \sigma_I) := (-i/\sqrt{2})(I, \tau_I)$ with τ_I being the Pauli matrices. We also define the symbol $\sigma_{IA}{}^B$ (which is called the $SU(2)$ soldering form in [1]) by using $n^{AA'} = n^i \sigma_i^{AA'}$ as $\sigma_{IA}{}^B := -\sqrt{2}i \sigma_{IAA'} n^{BA'} = (i/\sqrt{2})(\tau_I)_{AB}$.

where $\mathcal{A}_a^I := -2A^{(+)}_0{}^I{}_a$ and $(\tilde{\pi}^{(I)}{}_A{}^a, {}^+\tilde{\pi}^a)$ are defined by ⁶

$$\tilde{\pi}^{(I)}{}_A{}^a := \frac{\delta \mathcal{L}^{(+)}}{\delta \dot{\psi}^{(I)A}{}_a} = -\sqrt{2}i \epsilon^{abc} E_c^I \bar{\psi}^{(I)A'}{}_b \sigma_{IAA'}, \quad (4.3)$$

$${}^+\tilde{\pi}^a := \frac{\delta \mathcal{L}^{(+)}}{\delta \dot{A}_a} = \tilde{\pi}^a + i \tilde{B}^a \quad (4.4)$$

with

$$\begin{aligned} \tilde{\pi}^a := & \frac{e}{2N^2} q^{ab} \{ 2 (F_{tb} - N^d F_{db}) \\ & - \sqrt{2} (\bar{\psi}_t^{(I)} \psi_b^{(J)} - N^d \bar{\psi}_d^{(I)} \psi_b^{(J)}) \epsilon^{(I)(J)} \} \\ & - \frac{i}{2\sqrt{2}} \epsilon^{abc} \bar{\psi}_b^{(I)} \gamma_5 \psi_c^{(J)} \epsilon^{(I)(J)}, \end{aligned} \quad (4.5)$$

$$\tilde{B}^a = \frac{1}{2} \epsilon^{abc} F_{bc}. \quad (4.6)$$

In Eq. (4.5) the Majorana spinors $\psi_\mu^{(I)}$ are used for simplicity. On the other hand, the constraints are obtained from the variation of $\mathcal{L}^{(+)}$ with respect to Lagrange multipliers. Here we raise, in particular, the Gauss, $U(1)$ gauge, right-handed SUSY and left-handed SUSY constraints expressed by the canonical variables as follows; namely, ⁷

$$\mathcal{G}_I := \frac{\delta \mathcal{L}^{(+)}}{\delta \Lambda_t^I} = \mathcal{D}_a \tilde{E}_I^a - \frac{i}{\sqrt{2}} \tilde{\pi}^{(I)}{}_A{}^a \sigma_{IAB} \psi^{(I)B}{}_a = 0, \quad (4.7)$$

$$g := \frac{\delta \mathcal{L}^{(+)}}{\delta A_t} = \partial_a {}^+\tilde{\pi}^a + \lambda \psi^{(I)A}{}_a \tilde{\pi}^{(J)}{}_A{}^a \epsilon^{(I)(J)} = 0, \quad (4.8)$$

$$\begin{aligned} {}^R\mathcal{S}_A^{(I)} := & \frac{\delta \mathcal{L}^{(+)}}{\delta \psi^{(I)A}{}_t} = \mathcal{D}_a \tilde{\pi}^{(I)}{}_A{}^a + \frac{1}{\sqrt{2}} {}^+\tilde{\pi}^a \psi^{(J)B}{}_a \epsilon_{AB} \epsilon^{(I)(J)} \\ & + \lambda (2i \tilde{E}_I^a \sigma_{AB} \psi^{(I)B}{}_a - \tilde{\pi}^{(J)}{}_A{}^a A_a \epsilon^{(I)(J)}) = 0, \end{aligned} \quad (4.9)$$

$$\begin{aligned} {}^L\mathcal{S}_A^{(I)} := & \frac{\delta \mathcal{L}^{(+)}}{\delta \rho^{(I)A}{}_t} \\ = & -\sqrt{2} \tilde{E}_I^a \tilde{E}_J^b (\sigma^I \sigma^J)_A{}^B \left[2(\mathcal{D}_{[a} \psi^{(I)C}{}_{b]} - \lambda A_{[a} \psi^{(J)C}{}_{b]} \epsilon^{(I)(J)}) \epsilon_{BC} \right. \end{aligned}$$

⁶ The derivative for fermionic variables is treated as the left derivative unless stated otherwise.

⁷ We note that the ${}^+\tilde{\pi}^a$ appears in Eqs. (4.8), (4.9) and (4.10). If we use $(F_{\mu\nu})^2$ as the Maxwell kinetic term in Eq. (2.1), the $\tilde{\pi}^a$ (and not ${}^+\tilde{\pi}^a$) will appear in Eq. (4.8), and it is not possible to rewrite the canonical constraints in terms of the graded variables.

$$\begin{aligned}
& + \frac{i}{\sqrt{2}} \lambda \epsilon_{abc} \tilde{\pi}^{(I)}{}^B{}_c \Big] \\
& + \frac{i}{2} E^{-2} \epsilon_{def} \epsilon_{agh} (\sigma^I \sigma^J \sigma^K \sigma^L)_A{}^B \epsilon^{(I)(J)} \tilde{E}_I^e \tilde{E}_J^f \tilde{E}_K^g \tilde{E}_L^h \tilde{\pi}^{(J)}{}^B{}_d \\
& \times \left[\epsilon^{abc} \left\{ F_{bc} + \frac{1}{\sqrt{2}} \epsilon_{CD} (\psi^{(K)C}{}_b \psi^{(L)D}{}_c) \epsilon^{(K)(L)} \right\} + i \tilde{\pi}^a \right] = 0, \quad (4.10)
\end{aligned}$$

where the Lagrange multipliers, Λ_t^I and $\rho^{(I)A}{}_t$, are defined by

$$\Lambda_t^I := -2A^{(+)}{}_0{}^I{}_t, \quad \rho^{(I)A}{}_t := E^{-1} \overline{\psi}_{A't}^{(I)} n^{AA'}, \quad (4.11)$$

and the covariant derivatives on Σ are

$$\begin{aligned}
\mathcal{D}_a \tilde{E}_I^a &:= \partial_a \tilde{E}_I^a + i \epsilon_{IJK} \mathcal{A}_a{}^J \tilde{E}^{Ka}, \\
\mathcal{D}_a \tilde{\pi}^{(I)}{}^A{}_a &:= \partial_a \tilde{\pi}^{(I)}{}^A{}_a - \frac{i}{\sqrt{2}} \mathcal{A}_A{}^B{}_a \tilde{\pi}^{(I)}{}^B{}_a. \quad (4.12)
\end{aligned}$$

The left-handed SUSY constraint of Eq. (4.10) is not polynomial because of the factor E^{-2} , but the rescaled $E^2({}^L\mathcal{S}_A^{(I)})$ becomes polynomial because

$$E^2 = \frac{1}{6} \epsilon_{abc} \epsilon^{IJK} \tilde{E}_I^a \tilde{E}_J^b \tilde{E}_K^c. \quad (4.13)$$

The above canonical constraints (except for the Gauss constraint) have expressions different from those for the $N = 2$ 2-form SUGRA [8, 9]. This seems to originate from difference in the definition of momentum variables, in particular the momentum conjugate to the Maxwell field.

Now, by using the non-vanishing Poisson brackets⁸ among the canonical variables,

$$\{\mathcal{A}_a^I(x), \tilde{E}_J^b(y)\} = \delta_J^I \delta_a^b \delta^3(x - y),$$

⁸The Poisson brackets are defined for canonical variables (q^i, \tilde{p}_i) by using the right and left derivatives as $\{F, G\} := \int d^3z [(\delta^R F / \delta q^i(z))(\delta^L G / \delta \tilde{p}_i(z)) - (-1)^{|i|} (\delta^R F / \delta \tilde{p}_i(z))(\delta^L G / \delta q^i(z))]$ with $|i| = 0$ for an even (commuting) q^i while $|i| = 1$ for an odd (anticommuting) q^i .

$$\begin{aligned}
\{\psi^{(I)A}{}_a(x), \tilde{\pi}^{(J)}{}_B{}^b(y)\} &= -\delta^{(I)(J)}\delta_B^A\delta_a^b\delta^3(x-y), \\
\{A_a(x), {}^+\tilde{\pi}^b(y)\} &= \delta_a^b\delta^3(x-y),
\end{aligned} \tag{4.14}$$

we show that the Gauss, $U(1)$ gauge and right-handed SUSY constraints of Eqs. (4.7)-(4.9) form the graded algebra, $G^2SU(2)$. In fact, if we define the smeared functions,

$$\begin{aligned}
\mathcal{G}_I[\Lambda^I] &:= \int_{\Sigma} d^3x \Lambda^I \mathcal{G}_I, \\
g[a] &:= \int_{\Sigma} d^3x a g, \\
{}^R\mathcal{S}_A^{(I)}[\xi^{(I)A}] &:= \int_{\Sigma} d^3x \xi^{(I)A} {}^R\mathcal{S}_A^{(I)}
\end{aligned} \tag{4.15}$$

for convenience of the calculation, the Poisson brackets of \mathcal{G}_I, g and ${}^R\mathcal{S}_A^{(I)}$ are obtained as

$$\begin{aligned}
\{\mathcal{G}_I[\Lambda^I], \mathcal{G}_J[\Gamma^J]\} &= \mathcal{G}_I[\Lambda'^I], \\
\{\mathcal{G}_I[\Lambda^I], g[a]\} &= 0 = \{g[a], g[b]\}, \\
\{\mathcal{G}_I[\Lambda^I], {}^R\mathcal{S}_A^{(I)}[\xi^{(I)A}]\} &= {}^R\mathcal{S}_A^{(I)}[\xi'^{(I)A}], \\
\{g[a], {}^R\mathcal{S}_A^{(I)}[\xi^{(I)A}]\} &= \lambda {}^R\mathcal{S}_A^{(I)}[\xi''^{(I)A}], \\
\{{}^R\mathcal{S}_A^{(I)}[\xi^{(I)A}], {}^R\mathcal{S}_B^{(J)}[\eta^{(J)B}]\} &= \lambda \mathcal{G}_I[\Lambda'''^I] + g[a']
\end{aligned} \tag{4.16}$$

with the parameters, $\Lambda'^I, \Lambda'''^I, \xi'^{(I)A}, \xi''^{(I)A}$ and a' , being defined as

$$\begin{aligned}
\Lambda'^I &:= i \epsilon^{IJK} \Lambda_J \Gamma_K, \\
\Lambda'''^I &:= 2i \xi^{(I)A} \eta^{(J)B} \sigma^I{}_{AB} \delta^{(I)(J)}, \\
\xi'^{(I)A} &:= \frac{i}{\sqrt{2}} \Lambda^I \xi^{(I)B} \sigma_{IB}{}^A, \\
\xi''^{(I)A} &:= -a \xi^{(J)A} \epsilon^{(I)(J)}, \\
a' &:= \frac{1}{\sqrt{2}} \xi^{(I)A} \eta^{(J)B} \epsilon_{AB} \epsilon^{(I)(J)}.
\end{aligned} \tag{4.17}$$

The algebra of Eq. (4.16) coincides with the graded algebra, $G^2SU(2)$, which was first introduced in [10] in the framework of the BF theory.

5. The graded variables associated with the $G^2SU(2)$ algebra

In $N = 1$ chiral SUGRA, the graded variables of $GSU(2)$ was introduced based on the graded algebra which is satisfied by the Gauss and right-handed SUSY constraints [3, 4]. These graded variables simplify the expressions of all the canonical constraints, and therefore it becomes easier to find exact solutions of quantum constraints [4].

In order to introduce the graded variables of $G^2SU(2)$ in $N = 2$ chiral SUGRA, let us define the generators

$$J_{\hat{i}} := (J_I, J_\alpha, J_8), \quad (5.1)$$

which satisfy the same algebra as that of the constraints

$$C_{\hat{i}} := (\mathcal{G}_I, {}^R\mathcal{S}_\alpha, g_8), \quad (5.2)$$

where $({}^R\mathcal{S}_\alpha, g_8)$ stand for $({}^R\mathcal{S}_A^{(I)}, g)$, and the index \hat{i} runs over $(I, \alpha, 8)$ with $\alpha := (I)A$. Namely, we suppose that the $J_{\hat{i}}$ satisfy the $G^2SU(2)$ algebra

$$[J_{\hat{i}}, J_{\hat{j}}] = f_{\hat{i}\hat{j}}^{\hat{k}} J_{\hat{k}} \quad (5.3)$$

with $f_{\hat{i}\hat{j}}^{\hat{k}}$ being the structure constant determined from Eq. (4.16). The fundamental representation of this algebra is given by

$$J_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad J_2 = \frac{1}{2} \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad J_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{aligned}
J_1^{(1)} &= \sqrt{\frac{\lambda}{\sqrt{2}}} i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & J_2^{(1)} &= \sqrt{\frac{\lambda}{\sqrt{2}}} i \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
J_1^{(2)} &= \sqrt{\frac{\lambda}{\sqrt{2}}} i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, & J_2^{(2)} &= \sqrt{\frac{\lambda}{\sqrt{2}}} i \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \\
J_8 &= \lambda \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.
\end{aligned} \tag{5.4}$$

The supertrace [18] of the bilinear forms, $\text{STr}(J_i J_j)$, is given by

$$\begin{aligned}
\text{STr}(J_I J_J) &= \frac{1}{2} \delta_{IJ}, \quad \text{STr}(J_\alpha J_\beta) = \text{STr}(J_A^{(I)} J_B^{(J)}) = \sqrt{2} \lambda \epsilon_{AB} \delta^{(I)(J)}, \\
\text{STr}(J_8 J_8) &= 2\lambda^2, \quad \text{STr}(J_I J_\alpha) = 0 = \text{STr}(J_I J_8) = \text{STr}(J_\alpha J_8),
\end{aligned} \tag{5.5}$$

where the supertrace for a $G^2 SU(2)$ matrix M is defined by $\text{STr}(M) = M_{11} + M_{22} - M_{33} - M_{44}$. We introduce the metric $g_{\hat{i}\hat{j}}$ for $G^2 SU(2)$, which is of block-diagonal form, by

$$g_{\hat{i}\hat{j}} := 2 \text{STr}(J_i J_j) = (\delta_{IJ}, 2\sqrt{2}\lambda\epsilon_{AB}\delta^{(I)(J)}, 4\lambda^2). \tag{5.6}$$

Here we normalize $g_{\hat{i}\hat{j}}$ so that the condition $g_{IJ} = \delta_{IJ}$ be satisfied. The inverse $g^{\hat{i}\hat{j}}$ is given by

$$g^{\hat{i}\hat{j}} = \left(\delta^{IJ}, -\frac{1}{2\sqrt{2}\lambda} \epsilon^{AB} \delta^{(I)(J)}, \frac{1}{4\lambda^2} \right). \tag{5.7}$$

We shall lower or raise the index \hat{i} by using $g_{\hat{i}\hat{j}}$ and $g^{\hat{i}\hat{j}}$. For example, the $J^{\hat{i}}$ with upper index \hat{i} is defined by $J^{\hat{i}} := g^{\hat{i}\hat{j}} J_{\hat{j}}$.

As is seen from Eq. (4.2), the sets of the fields,

$$\mathcal{A}^{\hat{i}}_a := (\mathcal{A}^I_a, \psi^\alpha_a := \psi^{(I)A}_a, A^8_a := A_a), \tag{5.8}$$

$$\tilde{\mathcal{E}}^{\hat{i}}_a := (\tilde{E}^a_I, -\tilde{\pi}^a_\alpha := -\tilde{\pi}^{(I)}_{A^a}, +\tilde{\pi}^a_8 := +\tilde{\pi}^a), \tag{5.9}$$

play the role of the coordinate variables and their conjugate momenta, respectively. We shall refer to $\mathcal{A}_a^{\hat{i}}$ and $\tilde{\mathcal{E}}_i^a$ as the graded connection and the graded momentum, respectively.

Now let us define the graded variables \mathcal{A}_a and $\tilde{\mathcal{E}}^a$:

$$\mathcal{A}_a := \mathcal{A}_a^{\hat{i}} J_{\hat{i}}, \quad \tilde{\mathcal{E}}^a := \tilde{\mathcal{E}}_i^a J^{\hat{i}}. \quad (5.10)$$

Then the kinetic terms of Eq. (4.2) are expressed as a simple form, $2 \text{STr}(\tilde{\mathcal{E}}^a \dot{\mathcal{A}}_a)$, due to the relation $\text{STr}(J^{\hat{i}} J_{\hat{j}}) = (1/2) \delta_{\hat{j}}^{\hat{i}}$. Furthermore, we can show that the divergence of $\tilde{\mathcal{E}}^a$ can be rewritten as

$$\begin{aligned} \mathcal{D}_a \tilde{\mathcal{E}}^a &:= \partial_a \tilde{\mathcal{E}}^a + [\mathcal{A}_a, \tilde{\mathcal{E}}^a] \\ &= \partial_a \tilde{\mathcal{E}}_i^a J^{\hat{i}} + \mathcal{A}_a^{\hat{i}} \tilde{\mathcal{E}}_{\hat{j}}^a [J_{\hat{i}}, J^{\hat{j}}] \\ &= C_{\hat{i}} J^{\hat{i}}. \end{aligned} \quad (5.11)$$

Therefore, the Gauss, $U(1)$ gauge and right-handed SUSY constraints are unified into the $G^2 SU(2)$ Gauss law

$$\mathcal{D}_a \tilde{\mathcal{E}}^a = 0. \quad (5.12)$$

The left-handed SUSY constraint can also be expressed by using the graded variables, $\mathcal{A}_a^{\hat{i}}$ and $\tilde{\mathcal{E}}_i^a$ as in the case of $N = 1$ chiral SUGRA [4]. Indeed, ${}^L \mathcal{S}_A^{(I)}$ of Eq. (4.10) can be rewritten as

$$\begin{aligned} {}^L \mathcal{S}_A^{(I)} &= \lambda^{-1} (C_A^{(I) \hat{i} \hat{j} \hat{k}} \epsilon_{abc} \tilde{\mathcal{E}}_{\hat{i}}^a \tilde{\mathcal{E}}_{\hat{j}}^b \\ &\quad + \lambda^{-1} E^{-2} C_A^{(I) \hat{l} \hat{m} \hat{n} \hat{i} \hat{j} \hat{k}} \epsilon_{def} \epsilon_{cgh} \tilde{\mathcal{E}}_{\hat{l}}^e \tilde{\mathcal{E}}_{\hat{m}}^f \tilde{\mathcal{E}}_{\hat{n}}^g \tilde{\mathcal{E}}_{\hat{i}}^h \tilde{\mathcal{E}}_{\hat{j}}^d) (\mathcal{B}_{\hat{k}}^c \pm 2i \lambda^2 \tilde{\mathcal{E}}_{\hat{k}}^c) \\ &= 0, \end{aligned} \quad (5.13)$$

where $C_A^{(I) \hat{i} \hat{j} \hat{k}}$ and $C_A^{(I) \hat{l} \hat{m} \hat{n} \hat{i} \hat{j} \hat{k}}$ are defined by

$$C_A^{(I) I J \alpha} := -\frac{1}{2} (\sigma^I \sigma^J)_A{}^B \delta^{(I)(J)}, \quad (5.14)$$

$$C_A^{(I) LMNI\alpha 8} := -\frac{i}{4}(\sigma^L \sigma^M \sigma^N \sigma^I)_A{}^B \epsilon^{(I)(J)} \quad (5.15)$$

while all other components vanish, and the signatures \pm in the parenthesis of Eq. (5.13) are taken to be $+$ for $\hat{k} = I, 8$ while $-$ for $\hat{k} = \alpha$. In Eq. (5.13) we have also defined $\mathcal{B}^{\hat{i}a}$ as $\mathcal{B}^{\hat{i}a} = (1/2)\epsilon^{abc}\mathcal{F}^{\hat{i}}_{bc}$ with

$$\begin{aligned} \mathcal{F}^{\hat{i}}_{ab} &:= 2 \partial_{[a}\mathcal{A}^{\hat{i}}_{b]} + f_{\hat{j}\hat{k}}^{\hat{i}} \mathcal{A}^{\hat{j}}_a \mathcal{A}^{\hat{k}}_b \\ &= \left(\mathcal{F}^I_{ab} + 2i\lambda\psi^{(I)A}{}_{[a}\sigma^I{}_{|AB|}\psi^{(I)B}{}_{b]}, \right. \\ &\quad \left. 2(\mathcal{D}_{[a}\psi^{(J)A}{}_{b]} - \lambda A_{[a}\psi^{(J)A}{}_{b]})\epsilon^{(I)(J)}, \right. \\ &\quad \left. F_{ab} + \frac{1}{\sqrt{2}}\epsilon_{AB}(\psi^{(I)A}{}_{[a}\psi^{(J)B}{}_{b]})\epsilon^{(I)(J)} \right), \end{aligned} \quad (5.16)$$

where $\mathcal{F}^I_{ab} := 2 \partial_{[a}\mathcal{A}^I_{b]} + i \epsilon^I{}_{JK}\mathcal{A}^J_a \mathcal{A}^K_b$.

6. Quantization in the graded-connection representation

In this section, we consider the canonical quantization of $N = 2$ chiral SUGRA in the graded-connection representation: Namely, quantum states are represented by wavefunctionals $\Psi[\mathcal{A}]$, and operators of the graded variables $(\hat{\mathcal{A}}^{\hat{i}}_a, \hat{\mathcal{E}}^{\hat{i}a})$ act on $\Psi[\mathcal{A}]$ as

$$\hat{\mathcal{A}}^{\hat{i}}_a \Psi[\mathcal{A}] = \mathcal{A}^{\hat{i}}_a \Psi[\mathcal{A}], \quad \hat{\mathcal{E}}^{\hat{i}a} \Psi[\mathcal{A}] = \mp i \frac{\delta}{\delta \mathcal{A}^{\hat{i}}_a} \Psi[\mathcal{A}], \quad (6.1)$$

where the signatures in front of the derivative $(\delta/\delta \mathcal{A}^{\hat{i}}_a)$ are taken to be $-$ for $\hat{i} = I, 8$ while $+$ for $\hat{i} = \alpha$. In $N = 1$ chiral SUGRA [4], there are two main results about solutions of quantum constraints in the graded-connection representation of $GSU(2)$ as in the case of pure gravity [19, 20]. One is that with the factor-ordering of the triads to the right Wilson loops of the graded connection are annihilated by the quantum Gauss and right-handed SUSY constraints, and also annihilated by the quantum left-handed SUSY constraint for smooth loops (which do not have kinks

or intersections). The other is that with ordering the triads to the left in the left-handed SUSY constraint the exponential of the Chern-Simons form built with the graded connection solves all quantum constraints with a cosmological constant.

Let us examine quantum constraints for the above two cases in $N = 2$ chiral SUGRA, for which the left-handed SUSY constraint of Eq. (5.13) involves a nonpolynomial factor E^{-2} like the Hamiltonian constraint in the Einstein-Maxwell theory in the Ashtekar variable [17]. Firstly, we consider Wilson loops of the graded connection for $G^2SU(2)$,

$$W_\gamma[\mathcal{A}] = \text{STr} \left[P \exp \left(\oint_\gamma dy^a \mathcal{A}_a(y) \right) \right], \quad (6.2)$$

where the path ordered exponential is used and γ denotes loops on Σ . The Wilson loops of Eq. (6.2) are $G^2SU(2)$ invariant so that they are annihilated by the quantum $G^2SU(2)$ Gauss law (5.12), and for smooth loops they are also annihilated by the rescaled left-handed SUSY constraint, $\hat{E}^2({}^L\hat{\mathcal{S}}_A^{(I)})$. However, if the rescaled constraint is to be equivalent with the original constraint, then Eq. (6.2) fails to be solutions in $N = 2$ chiral SUGRA as stated in [17] since the operator \hat{E}^2 annihilates $W_\gamma[\mathcal{A}]$.

Secondly, we consider the exponential of the Chern-Simons form built with the graded connection of $G^2SU(2)$,

$$\Psi_{\text{CS}}[\mathcal{A}] = \exp \left[-\frac{1}{2\lambda^2} \int_\Sigma d^3x \epsilon^{abc} \text{STr} \left(\mathcal{A}_a \partial_b \mathcal{A}_c + \frac{2}{3} \mathcal{A}_a \mathcal{A}_b \mathcal{A}_c \right) \right]. \quad (6.3)$$

This is an exact state functional that solves all quantum constraints of $N = 2$ chiral SUGRA: In fact, Eq. (6.3) is annihilated by the $G^2SU(2)$ Gauss law because of a $G^2SU(2)$ invariance for $\Psi_{\text{CS}}[\mathcal{A}]$, while it is annihilated by the quantum left-handed SUSY constraint for Eq. (5.13) with the factor-ordering of the triads to the left;

namely,

$${}^L\hat{\mathcal{S}}_A^{(I)}\Psi_{\text{CS}}[\mathcal{A}] = \dots \times \left(\mathcal{B}_{\hat{k}}{}^c + 2\lambda^2 \frac{\delta}{\delta \mathcal{A}_{\hat{k}}^c} \right) \Psi_{\text{CS}}[\mathcal{A}] = 0, \quad (6.4)$$

since $(\delta/\delta \mathcal{A}_{\hat{k}}^c)\Psi_{\text{CS}}[\mathcal{A}] = (-1/2\lambda^2)\mathcal{B}_{\hat{k}}{}^c\Psi_{\text{CS}}[\mathcal{A}]$. The $\Psi_{\text{CS}}[\mathcal{A}]$ of Eq. (6.3) coincides with the $N = 2$ supersymmetric Chern-Simons solution obtained in [8, 10].

7. Conclusions

In this paper we have reconstructed the Ashtekar's canonical formulation of $N = 2$ SUGRA starting from the $N = 2$ chiral Lagrangian derived by closely following the method employed in the usual SUGRA. We have modified the Maxwell kinetic term as $(F_{\mu\nu}^{(-)})^2$ in the globally $O(2)$ invariant Lagrangian obtained in [14]. In addition we have gauged $O(2)$ invariance of the Lagrangian so that we have obtained the full graded algebra, $G^2SU(2)$, of the Gauss, $U(1)$ gauge and right-handed SUSY constraints in the canonical formulation. The left-handed SUSY constraint has the nonpolynomial factor E^{-2} as in the case of the Einstein-Maxwell theory in the Ashtekar variable [17].

We have introduced the graded variables $(\mathcal{A}_a, \tilde{\mathcal{E}}^a)$ associated with the $G^2SU(2)$ algebra and showed that the $G^2SU(2)$ Gauss law, $\mathcal{D}_a\tilde{\mathcal{E}}^a = 0$, coincides with the Gauss, $U(1)$ gauge and right-handed SUSY constraints. We have also rewritten the left-handed SUSY constraint in terms of the graded variables. Based on the representation in which the graded connection is diagonal, we have examined the solutions of quantum constraints obtained in $N = 1$ chiral SUGRA [4]; namely, Wilson loops of the graded connection, $W_\gamma[\mathcal{A}]$, and the exponential of the Chern-Simons form built with the graded connection, $\Psi_{\text{CS}}[\mathcal{A}]$. If the left-handed SUSY constraint rescaled by E^2 is to be equivalent with the original constraint, then the Wilson loops of $W_\gamma[\mathcal{A}]$ fail to be solutions in $N = 2$ chiral SUGRA since the operator

\hat{E}^2 annihilates $W_\gamma[\mathcal{A}]$. On the other hand, the exponential of the Chern-Simons form, $\Psi_{\text{CS}}[\mathcal{A}]$, is an exact state functional that solves all quantum constraints of $N = 2$ chiral SUGRA. This solution was first derived in [8] in the 2-form SUGRA, and later given in [10] based on the BF theory. In this paper we have obtained the same result starting from the chiral Lagrangian which is closely related to the usual SUGRA.

The extension to higher N SUGRA is now investigated.

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